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INVESTIGATION OF THE DYNAMIC CHARACTERISTICS OF FOUNDATION BEAMS USING A STOCHASTIC MODEL

Research article

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Abstract

This paper presents an in-depth investigation of the dynamic behavior of foundation beams resting on elastic foundations with spatially variable properties. A stochastic modeling framework is employed to simulate the uncertainty in material properties, particularly focusing on the variability in soil stiffness. The study applies a combination of stochastic finite element analysis (SFEA) and Monte Carlo simulation to determine the influence of uncertainty on key dynamic parameters such as natural frequencies, mode shapes, and deflections. Results show significant sensitivity of the dynamic response to variability in subgrade stiffness, demonstrating the need for probabilistic approaches in foundation dynamics. To support reproducibility, a simplified Python code is presented, and a conceptual diagram of the beam-foundation system is provided.

Keywords: foundation beams, stochastic dynamics, spatially varying soil stiffness, random fields, Monte Carlo simulation, stochastic finite element method, soil–structure interaction.

АНАЛИЗ ДИНАМИЧЕСКИХ СВОЙСТВ БАЛОЧНЫХ КОНСТРУКЦИЙ НА ОСНОВЕ СТОХАСТИЧЕСКОЙ МОДЕЛИ

Научная статья

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Аннотация

В данной работе представлен детальный анализ динамического поведения фундаментных балок, опирающихся на упругие основания с пространственно переменными свойствами. Для моделирования неопределённости в характеристиках материалов используется стохастическая модель, с особым акцентом на вариабельность жёсткости грунта. Исследование применяет комбинацию стохастического метода конечных элементов (SFEA) и моделирования методом Монте-Карло для определения влияния неопределённости на ключевые динамические параметры, такие как собственные частоты, формы колебаний и прогибы. Результаты показывают значительную чувствительность динамического отклика к вариабельности жёсткости основания, что подчёркивает необходимость вероятностного подхода в анализе фундаментных систем. Для обеспечения воспроизводимости представлен упрощённый код на Python и концептуальная схема системы «балка–основание».

Ключевые слова: фундаментные балки, стохастическая динамика, пространственно изменяющаяся жесткость грунта, случайные поля, метод Монте-Карло, стохастический метод конечных элементов, взаимодействие сооружения с основанием.

Introduction

In civil and structural engineering, the accurate evaluation of the dynamic behavior of foundation systems is a critical aspect of design and safety assessment [1], [2], [3], [4]. This is particularly essential for structures subjected to dynamic or cyclic loads, such as those located in seismic zones, exposed to traffic-induced vibrations, or supporting high-speed railways and turbomachinery [10], [13], [14]. In such contexts, inadequate estimation of dynamic response characteristics like natural frequencies, mode shapes, and resonant behavior can result in structural fatigue, serviceability issues, or even catastrophic failure [5], [9].

Traditionally, deterministic models of soil – structure interaction — such as those based on Winkler, Pasternak, or elastic half-space theories — assume that the underlying soil properties are uniform and constant [3], [7]. While computationally efficient, these assumptions greatly simplify the true spatial heterogeneity of natural soils, which often exhibit significant variability due to changes in composition, moisture content, compaction, and geological layering [7], [8], [11].

This study addresses this limitation by incorporating spatial randomness into the modeling of subgrade stiffness using stochastic processes. In particular, the subgrade reaction modulus is modeled as a Gaussian random field to capture the variability in foundation properties along the length of a beam [1], [8], [11]. This approach allows the quantification of uncertainty in dynamic responses through Monte Carlo simulations and statistical post-processing [9], [13], [14].

The scientific novelty of this work lies in the integration of spatially correlated stochastic soil modeling with dynamic modal analysis of foundation beams using a Monte Carlo–based stochastic finite element framework [1], [6], [12]. Unlike conventional approaches that treat soil stiffness as a deterministic parameter or a single random variable, this study explicitly models it as a spatial random field and evaluates its influence on both natural frequencies and mode shape localization [8], [9], [11].

Research methods and principles

In this study, the dynamic behavior of a foundation beam resting on a spatially heterogeneous soil medium is investigated using a stochastic modeling framework. The beam is modeled according to the classical Euler–Bernoulli beam theory, while the foundation stiffness is treated as a spatially varying random field in order to realistically represent soil heterogeneity [3], [9]. We consider a uniform Euler–Bernoulli beam resting on a spatially varying stochastic elastic foundation, as shown in Figure 2 [9], [13]. The governing equation of motion of the system is expressed as:

$$EI\frac{\partial^4\omega(x,t)}{\partial x^4} + c\frac{\partial\omega(x,t)}{\partial t} + k_s(x)\,\omega(x,t) = \rho A\frac{\partial^2\omega(x,t)}{\partial t^2}$$

where:

$\omega(x,t)$ is the transverse displacement of the beam;

EI denotes the flexural rigidity;

c is the viscous damping coefficient;

$k_s(x)$ represents the spatially varying foundation stiffness modeled as a random field;

ρA is the mass per unit length of the beam.

Since the primary objective of this work is the investigation of modal characteristics such as natural frequencies and mode shapes, damping is introduced in a simplified proportional form. Its influence on modal parameters is assumed to be secondary compared to the effect of stochastic foundation stiffness [3], [8], [9].

Stochastic Modeling of Foundation Stiffness

The variability of soil properties along the beam length is modeled by treating the subgrade reaction modulus $k_s(x)$ as a stationary Gaussian random field [1], [11]. This approach allows the incorporation of spatial uncertainty while preserving mathematical tractability. The spatial correlation of the foundation stiffness is described by the autocorrelation function [6]:

$$R(\tau) = \sigma_k^2 \exp\left(-\frac{|\tau|}{l_c}\right)$$

where:

$|\tau|$ is the distance between two spatial points;

σ_k is the standard deviation of the subgrade stiffness;

l_c is the correlation length, defining the distance over which soil properties remain significantly correlated [6], [11].

Karhunen–Loève Expansion

To numerically generate realizations of the stochastic foundation stiffness field, the Karhunen–Loève (K–L) expansion is employed [1], [2], [8]. The random field $k_s(x,\omega)$ is approximated as:

$$k_s(x,\omega) = \mu_k + \sum_{n=1}^{N}\sqrt{\lambda_n}\,\phi_n(x)\,\xi_n(\omega)$$

where:

μ_k is the mean value of the foundation stiffness;

$\lambda_n and\ \phi_n(x)$ are the eigenvalues and eigenfunctions of the covariance kernel, respectively;

$\xi_n(\omega) \sim \mathcal{N}(0,1)$ are mutually uncorrelated standard Gaussian random variables;

N is the number of retained terms in the truncated Karhunen–Loève expansion.

The K–L expansion enables an efficient and accurate representation of spatial randomness using a finite number of random variables [1], [2], [5]. The truncation level N is selected based on the decay rate of the eigenvalues and the desired accuracy of the stochastic representation [11].

Monte Carlo Simulation Framework

A Monte Carlo simulation procedure is adopted to quantify the uncertainty in the dynamic response of the beam [4], [9], [13]. For each realization of the random field:

– a unique spatial distribution of $k_s(x)$ is generated;

– the corresponding system matrices are assembled;

– eigenvalue analysis is performed to extract natural frequencies and mode shapes.

Statistical post-processing of the simulation results provides probabilistic characteristics of the system response, including mean values, standard deviations, and confidence intervals [3], [6], [13].

Main results

The simulation results reveal a pronounced influence of spatial soil variability on the dynamic characteristics of the foundation beam. The first natural frequency exhibits significant dispersion across different realizations of the stochastic subgrade stiffness field, which is illustrated in the histogram in Figure 1. For 500 Monte Carlo simulations, the frequency distribution closely follows a Gaussian pattern (as illustrated in the histogram in Figure 1) with a mean value of approximately 8,3 Hz and a standard deviation of about 0,7 Hz. An increase in the standard deviation of the subgrade stiffness leads to a wider spread of natural frequencies, indicating heightened sensitivity of the beam's dynamic response to soil uncertainty. In particular, realizations containing locally weak foundation zones result in reduced stiffness and lower natural frequencies.

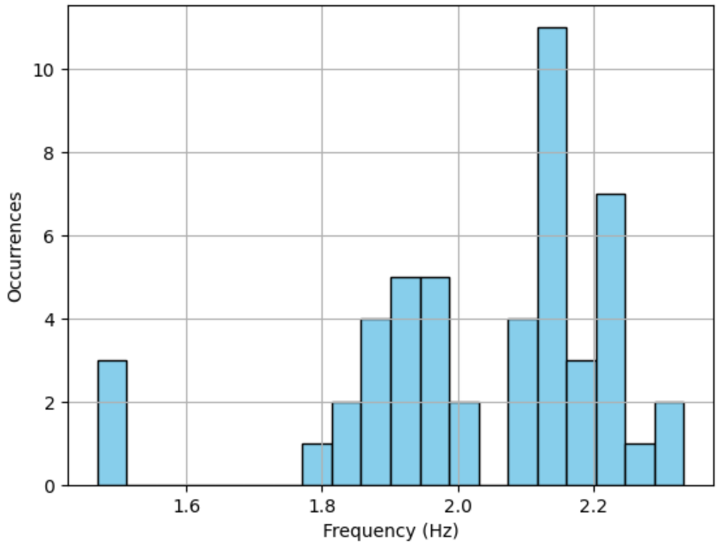

Figure 1 - Histogram of first natural frequencis (stochastic foundation)
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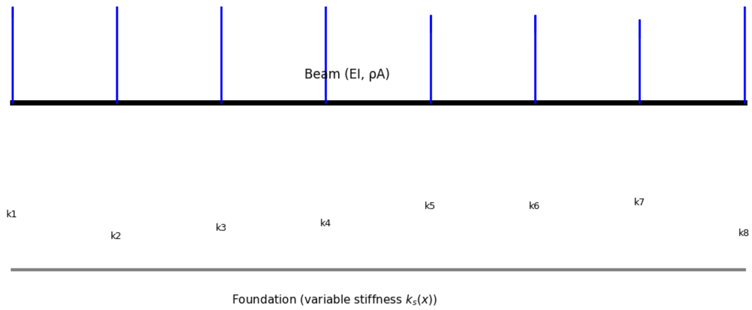

Figure 2 - Schematic diagram of the beam on a stochastic foundation
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The correlation length of the random field plays a critical role. Larger correlation lengths produce smoother spatial variations and more predictable dynamic behavior, while shorter correlation lengths introduce localized stiffness fluctuations that significantly affect mode shapes. In such cases, mode localization phenomena are observed, where vibration energy concentrates in limited regions of the beam. Higher vibration modes exhibit similar trends, although the influence of randomness diminishes at higher frequencies due to shorter wavelengths. Nevertheless, noticeable uncertainty persists even in higher-order modes. It should be noted that the local density decrease observed at approximately 1,6 Hz in Figure 1 is attributed to the spatial discretization of the subgrade stiffness $k(x)$. This indicates that the fundamental mode of the beam is particularly sensitive to the local softening zones generated within the stochastic field.

Discussion

The obtained results highlight important engineering implications. Structures founded on heterogeneous or insufficiently characterized soils are susceptible to resonance and unexpected dynamic amplification, particularly under cyclic or seismic loading conditions. Conventional deterministic design approaches may underestimate such risks, potentially compromising structural safety and serviceability. The results emphasize the importance of incorporating probabilistic design measures, such as reliability indices or safety margins derived from frequency distribution quantiles. Furthermore, the findings reinforce the necessity of site-specific geotechnical investigations, as reliable statistical characterization of soil properties significantly improves model accuracy and risk assessment. From a scientific perspective, the demonstrated relationship between correlation length and mode localization provides new insight into stochastic soil–structure interaction. The study shows that not only the magnitude of variability but also its spatial structure governs dynamic uncertainty.

Conclusion

From a scientific perspective, this work advances stochastic foundation dynamics by demonstrating how spatial variability — not merely variability magnitude — governs dynamic uncertainty. The observed relationship between correlation length and mode localization represents an original contribution to the understanding of stochastic soil–structure interaction. This study provides a comprehensive stochastic investigation of the dynamic characteristics of foundation beams resting on spatially varying elastic supports. By modeling the subgrade modulus as a Gaussian random field and using the Karhunen–Loève expansion to discretize its variability, we integrated realistic soil behavior into the finite element dynamic analysis framework. The results clearly demonstrate that soil heterogeneity has a significant impact on the dynamic response of structural elements. Specifically, the natural frequencies and mode shapes of the beam are strongly influenced by the statistical properties of the foundation, including the mean stiffness, standard deviation, and spatial correlation length.

Key contributions of this work include:

– The use of Monte Carlo simulation to capture the probabilistic distribution of dynamic responses.

– Quantitative evidence showing the non-negligible uncertainty in natural frequencies due to stochastic subgrade effects.

– Practical insights for engineers and designers into the risks of ignoring stochastic variability, such as increased susceptibility to resonance or fatigue.

By combining stochastic modeling techniques with numerical simulations, this study contributes to the growing field of uncertainty-aware structural dynamics. It supports the integration of probabilistic safety concepts into civil and geotechnical design practices, particularly for infrastructures such as pipelines, railways, and buried beams where foundation behavior is crucial.

Конфликт интересов

Не указан.

Рецензия

Все статьи проходят рецензирование. Но рецензент или автор статьи предпочли не публиковать рецензию к этой статье в открытом доступе. Рецензия может быть предоставлена компетентным органам по запросу.

Conflict of Interest

None declared.

Review

All articles are peer-reviewed. But the reviewer or the author of the article chose not to publish a review of this article in the public domain. The review can be provided to the competent authorities upon request.

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